Problem 2.3

(a) The quadratic and linear drag forces on a moving sphere in a fluid are given by (2.84) and (2.82) (Problems 2.4 and 2.2). Show that the ratio of these two kinds of drag force can be written as $f_{\text{quad}}/f_{\text{lin}} = R/48$,[9] where the dimensionless **Reynolds number** R is

$$R = \frac{Dv\varrho}{\eta} \tag{2.83}$$

where D is the sphere's diameter, v its speed, and ϱ and η are the fluid's density and viscosity. Clearly the Reynolds number is a measure of the relative importance of the two kinds of drag.[10] When R is very large, the quadratic drag is dominant and the linear can be neglected; vice versa when R is very small. **(b)** Find the Reynolds number for a steel ball bearing (diameter 2 mm) moving at 5 cm/s through glycerin (density 1.3 g/cm^3 and viscosity 12 N $\cdot$ s/m^2 at STP).

Solution

Part (a)

The quadratic drag force on a sphere in a fluid is given in Problem 2.4.

$$f_{\text{quad}} = \kappa\varrho A v^2 = \frac{1}{4}\varrho A v^2 \tag{2.84}$$

The linear drag force on a sphere in a fluid is given in Problem 2.2.

$$f_{\text{lin}} = 3\pi\eta D v \tag{2.82}$$

Therefore, the ratio of these two drag forces is [A is the cross-sectional area, $\pi r^2 = \pi(D/2)^2$]

$$\frac{f_{\text{quad}}}{f_{\text{lin}}} = \frac{\frac{1}{4}\varrho A v^2}{3\pi\eta D v} = \frac{\frac{1}{4}\varrho\left(\frac{1}{4}\pi D^2\right)v^2}{3\pi\eta D v} = \frac{1}{48}\frac{\rho D v}{\eta} = \frac{1}{48}R.$$

Part (b)

For a steel ball bearing with $D = 2$ mm moving through glycerin at $v = 5$ cm/s, the Reynolds number is

$$R = \frac{Dv\varrho}{\eta} = \frac{\left(2\text{ mm} \times \frac{1\text{ m}}{1000\text{ mm}}\right)\left(5\,\frac{\text{cm}}{\text{s}} \times \frac{1\text{ m}}{100\text{ cm}}\right)\left[1.3\,\frac{\text{g}}{\text{cm}^3} \times \frac{1\text{ kg}}{1000\text{ g}} \times \left(\frac{100\text{ cm}}{1\text{ m}}\right)^3\right]}{12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

$$\approx 0.01.$$

Since $R/48 \approx 0.0002 \ll 1$, the quadratic drag force is negligible compared to the linear drag force.

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[9]The numerical factor 48 is for a sphere. A similar result holds for other bodies, but the numerical factor is different for different shapes.

[10]The Reynolds number is usually defined by (2.83) for flow involving any object, with D defined as a typical linear dimension. One sometimes hears the claim that R is the ratio $f_{\text{quad}}/f_{\text{lin}}$. Since $f_{\text{quad}}/f_{\text{lin}} = R/48$ for a sphere, this claim would be better phrased as "R is roughly of the order of $f_{\text{quad}}/f_{\text{lin}}$."